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Following our recent finding [1] that for the final state of continued spherical gravitational collapse of sufficiently massive bodies, the final gravitational mass of the fluid,  $M_f \rightarrow 0$ , we show that for a physical fluid the eventual value of  $2M_f/R_f \rightarrow 1$  rather than  $2M_f/R_f < 1$  (the speed of light  $c = 1$  and the gravitational constant  $G = 1$ ), indicating the approach to a zero-mass black hole. We also point out that as the final state is approached the curvature components tend to blow up, also the proper radial distance  $l$  and the proper time (measured along a radial worldline)  $\tau \rightarrow \infty$  indicating that actually the singularity is never attained. We also identify that, the final state may correspond to the local 3-speed attaining either  $V = 0$  or  $V \rightarrow c$ , even though invariant circumference contraction speed  $U = dR/d\tau \rightarrow 0$ . Nonetheless, at a finite observation epoch, such Eternally Collapsing Objects (ECOs) may have a local speed of collapse  $V \ll c$  and the labframe speed of collapse may be negligible because of high surface gravitational redshift. However, if quantum back reaction in the strong gravity regime would cause a phase transition of the form, pressure  $p = -\rho$ , where  $\rho$  is the density of the collapsing fluid, it may be possible to have static Ultra Compact Objects (UCOs) of arbitrary high mass [2]. While supposed Black Holes have no intrinsic magnetic field, ECOs or UCOs are likely to possess strong intrinsic magnetic field and we point out that there are already some tentative evidence for existence of such intrinsic magnetic fields in some Black Hole Candidates [3,4]. For the benefit of the readers who may not have gone through Paper I, we also include here the summary of the same. It clearly shows that the central result of Paper I can be derived even without knowing the meaning of the nomenclature  $V$  or without imposing any of property of  $V$  such as whether  $V < 1$  or not. In addition we consolidate the same result from other physical considerations too.<sup>1</sup>

Key Words: Gravitational Collapse, Eternally Collapsing Object, Ultra Compact Object

## I. INTRODUCTION

One of the questions haunting the physics since Newton unravelled the universal attractive nature of Gravity is what would be the “final state” of spherical gravitational collapse of sufficiently massive bodies. As of now, it is generally believed that collapsing bodies whose eventual mass is more than  $\sim 4 - 5M_\odot$  (solar mass), must become black holes (BH). Here, it must be mentioned that, in a strict sense, it has never been possible to find any *exact* analytical solution of the collapse problem by invoking an exact equation of state (EOS) and radiation transport properties of the collapsing fluid. A deep introspection would show that, neither will it ever be possible to do so in a strictly rigorous manner. This is so because, in order to handle the collapse problem in a really rigorous manner, without invoking any preconceived notions, one would require to know the march of the *ever evolving* EOS and radiation transport properties all the way upto the singularity! One can claim that one needs to know the march of the EOS only upto the state of formation of the horizon because, it can be shown that once the collapsing matter is enshrouded by an Event Horizon (EH), collapse upto the central singularity is inevitable. But since the mass of the collapsing fluid ( $M$ ) is changing, so is the would be location of the EH ( $R_g = 2M$ ), and hence it is not known beforehand. Thus, in principle, one would indeed require to know the march of relevant parameters all the way upto the singularity in order to study the collapse problem in an exact fashion. At this juncture, one may recall that as far as analytical solutions are concerned, the faith in the inevitability of formation BHs or EHs essentially relies on the analytical solution of Oppenheimer and Snyder (1939, OS) [5]. OS considered a highly idealized case of collapse of a pressureless “dust” of uniform density. But we have already shown that, if interpreted correctly and consistently, their work shows that no trapped surface or Event Horizon (EH) is formed at a finite value of  $M$  [1]. Further, starting from the seventies, several authors have tried to semi-analytically examine the case of collapse of a relatively more realistic fluid - an “inhomogeneous dust”. These authors claim that the collapse of an inhomogeneous dust need not

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lead to the formation of an EH ! In other words, inhomogeneous collapse may lead to the formation of the so-called “naked singularities” [6]. Again, here, one may invoke the so-called “cosmic censorship conjecture” (CCC) of Penrose [7] which asserts that for realistic cases, collapse must lead to the formation of BHs rather than naked singularities. However, given an initial gravitational mass  $M_i$ , the running value of the gravitational mass,  $M$ , would be constantly changing until the EH is formed and CCC *can not tell at what value of  $M$ , the horizon would form*. Such questions can be hoped to be answered only by the solution of actual collapse equations, if it were possible. Thus, even if one accepts CCC for the time being, in order to rigourously study the problem of gravitational collapse, one indeed needs to know the evolving EOS for arbitrary high density and temperature of the collapsing fluid. Second, since there is no proof for CCC, and it being what it is, a conjecture, we realize that it is unrealistic to hope for any strictly rigourous solution of the problem of collapse by any numerical means.

And it is in this perspective that recently we have discovered an extremely important *global property* of the General Relativistic (GR) collapse equations for a spherically symmetric physical fluid which means that (i) no trapped surface is ever formed and if the fluid indeed undergoes continued collapse, the final state must have  $M_f \equiv 0$  as  $R_f = 0$  [1]. In view of the importance of this result and for the sake of those readers who may not have read Paper I, we present the salient feature of it below. After this we shall proceed to address the aspects listed in the abstract of this paper.

## II. CRUX OF PAPER I

By analyzing the GR collapse equations in a generic comoving coordinate system described by the interior metric:

$$ds^2 = g_{00}dt^2 + g_{rr}dr^2 + R(r,t)^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where  $\theta$  and  $\phi$  are angular coordinates,  $R$  is the circumference coordinate and  $N(r)$  is the total number of baryons inclosed within the sphere of constant coordinate radius  $r$ , we have shown in paper I [1], that

$$\frac{2M(r,t)}{R} \leq 1 \quad (2)$$

This result depends solely on the global properties of the field equations manifest through the inverse curvature like parameter:

$$\Gamma \equiv \frac{dR}{dl} \quad (3)$$

where  $dl = \sqrt{-g_{rr}}dr$  is an element of proper radial distance along the worldline and the contraction (or expansion) rate of the invariant circumference radius is [8–14]

$$U \equiv dR/d\tau, \quad (4)$$

Here  $d\tau = \sqrt{g_{00}}dt$  is an element of proper time measured by a local  $r = \text{constant}$  observer. Clearly these two parameters are interrelated through an equation

$$U = \Gamma V \quad (5)$$

where

$$V \equiv \frac{dl}{d\tau} = \frac{\sqrt{-g_{rr}}dr}{\sqrt{g_{00}}dt} \quad (6)$$

Though the actual collapse equations are extremely complicated and nonlinear and defy *exact* solutions either analytically or numerically, nevertheless, they are intertwined globally through an Eq. connecting [8–14]  $\Gamma$  and  $U$ :

$$\Gamma^2 = 1 + U^2 - \frac{2M}{R} \quad (7)$$

Eqs.(5) and (7) can be combined in a master global equation:

$$\Gamma^2(1 - V^2) = 1 - \frac{2M}{R} \quad (8)$$

Or,

$$\frac{1}{-g_{rr}}(\partial R/\partial r)^2(1 - V^2) = 1 - \frac{2M}{R} \quad (9)$$

Note that although  $\Gamma$  has a partial derivative nature with respect to  $r$ , it is a total derivative with respect to  $l$  [10,11] because the notion of a fixed  $t$  is implied in the definition of  $dl$ . Also, note that for radial worldlines with  $d\theta = d\phi = 0$ , Eq.(1) can be rewritten as [15]

$$ds^2 = dt^2 g_{00}[1 - V^2] \quad (10)$$

Since for material particles,  $ds^2 \geq 0$ , the foregoing Eq. demands that

$$g_{00}(1 - V^2) \geq 0 \quad (11)$$

Further since the determinant of the metric tensor  $g = R^4 \sin^2 \theta g_{rr} g_{00}$  is always negative [15],  $g \leq 0$ , it follows that

$$g_{rr} g_{00} \leq 0, \text{ or, } -g_{rr} g_{00} \geq 0 \quad (12)$$

By dividing the inequality (11) by (12), we obtain

$$\frac{1 - V^2}{-g_{rr}} \geq 0 \quad (13)$$

By feeding the inequality (13) in Eq.(9) we find that both sides of it are positive. Therefore, the collapse equations obey the most general constraint that

$$1 - \frac{2M}{R} \geq 0 \quad (14)$$

and from which Eq.(2) follows. This means that contrary to the intuitive idea [16], GTR actually does not allow formation of trapped surfaces (there is a popular misconception that the “singularity theorems” “prove” the existence of trapped surfaces, but actually, they *assume* the existence of the same). This same result can also be obtained by using the radiative Vaidya metric [17]. Further note that in order that Eq.(2) is really satisfied, under the assumption of positivity of mass [18], we must have

$$M \rightarrow 0 \text{ as } R \rightarrow 0 \quad (15)$$

which means that the final gravitational mass of any gravitational singularity must be zero,  $M_f = 0$ .

The fact  $M_f = 0$  means that the “gravitational mass defect”  $\rightarrow M_i c^2$ , the original mass-energy of the fluid. As is well known, as any self-gravitating isolated body contracts it becomes hotter and radiates energy (in the form of photons, neutrinos etc.) and this is known as Kelvin-Helmholtz process. This result is obtained by just combining classical Virial Theorem and total energy conservation. Thermodynamically, this happens because of negative specific heat associated with self-gravity. As the self-gravitating body contracts it becomes gravitationally more bound, i.e, its total energy decreases. Therefore it must radiate out in order to satisfy total energy conservation. In relativity loss of energy from the body means decrease of its gravitational mass  $M$ . Note that this loss/decrease of mass is not due to loss of physical matter from the body and the baryonic mass of the body,  $M_0$ , can remain fixed while  $M$  decreases steadily.

On the other hand if we focus our attention on the fixed baryonic mass of the system, it may indeed be possible to pack baryons sufficiently closely to achieve a state  $2M_0/R > 1$  and in fact to chase the limit  $2M_0/R \rightarrow \infty$ . In Newtonian physics the gravitational mass  $M \equiv M_0$ , and thus, Newtonian physics may admit of a BH. It is small wonder then that *the concept of a BH actually arose almost two hundred years ago* [19]. And the idea that the formation of a “trapped surface” with  $2GM/R > 1$  is most natural is thus deeply ingrained into the intuitive (Newtonian) notions which do not distinguish between baryonic and gravitational mass. In contrast, in GTR, the total mass energy  $M \sim M_0 + E_g + E_{\text{internal}} + E_{\text{dynamic}}$ ; where the gravitational energy is negative, and is some evolving nonlinear function of  $M$ ,  $E_g = -f(M, R)$ . In the limit of weak gravity  $f \sim GM^2/R$ , but as the collapse proceeds, the grip of self-gravity becomes tighter and  $M$  starts becoming reasonably smaller than  $M_0$  or  $M_i$ . And if it were possible for  $M$  to assume negative values, for continued gravitational collapse, the non-linear  $E_g$  would relentlessly push  $M \rightarrow -\infty$ . However, there are positive energy theorems [18], which state that the mass-energy of an *isolated*

body can not be negative,  $M \geq 0$ , and, physically, which means that, gravitation can never be *repulsive*. And hence, the continued collapse process comes to a *decisive end* with  $M_f = 0$ . Physically the state  $M = 0$  may occur when the energy liberated in the process (as measured by a distant inertial observer  $S_\infty$ ),  $Q \rightarrow M_i c^2$  because  $M_f = M_i - Q$ .

Further the present study would show that the final  $M_f = 0$  state corresponds to the formation of a horizon at  $R_f = 0$  with  $2GM_f/R_f = 1$ . And since horizon is formed at  $M_f = 0$  the system cannot lose further energy and its mass cannot become negative. Thus the result that horizon would be formed at  $M_f = 0$  substantiates the positive energy theorems. Our result that the assumption of formation of “trapped surfaces” is not realized confirms Einstein’s idea [20] that Schwarzschild singularities can not occur in practice. However, Einstein’s exercise was based on the non-collapsing scenarios and the theoretical formalism for GTR collapse including pressure was developed in the sixties, much after Einstein’s exercise.

One important thing to note here is that in the above derivation of Eq.(2) or (14), *we did not even require to know the meaning* of the nomenclature  $V$ , and neither did we require to impose any condition on the property of  $V$ , (e.g, whether  $V \leq 1$  or not).

### A. Physical Interpretation of $V$

Landau & Lifshitz (LL) [15] first defined local 3-speed as  $dl/d\tau$  for a “particle” moving in a “*constant*” gravitational field, i.e, when metric components do not depend on time “ $x^0$ ” or “ $t$ ” e.g., a particle moving around a spherical body. Some readers might think that, the concept of 3-speed, therefore, may not be extended to a dynamic case, i.e, when the metric components are functions of  $x^0$ . This is not true because (i) both  $dl$  and  $d\tau$  remain definable even when  $g_{ik}$ s depend on  $x^0$  [15] and (ii) then one would not even be able to define the energy momentum tensor of a fluid. For instance, in the Section entitled, “The centrally symmetric gravitational field” LL [15] discuss the field equations of a dynamic fluid involving local 3-speed (below eq. 100.20). In our case,  $V$  happens to be the local 3 speed of an fluid element at a particular  $r$  because of *implicit* dependence of  $r$  on  $t$ . At first sight it may appear that since  $r$  is a comoving coordinate,  $dr \equiv 0$ , and therefore  $V \equiv 0$ . Then how to extract the *implicit* dependence of  $r$  on  $t$ ? As explained in Paper I, it should be done in the following way:

Focus attention on a certain comoving observer at  $r = r$  at its comoving time  $t = t$ . Let this be the observer 1. There is an adjacent observer 2 at  $r = r + dr$  at  $t = t$  as recorded by clock 1. Take a snapshot of this scenario at a time  $t = t$  as recorded by clock 1. Now take another snapshot of the fluid layers at a time  $t = t + dt$  as recorded by the same clock 1. Now superimpose the two snapshots. Observer 1 would find that his present position (at  $t = t + dt$ ) is matching with the previous position held by clock 2 at  $r = r + dr$ . From this measurement, observer 1 would know that during the interval  $dt$  as recorded by himself, he has travelled a distance  $dr$  *with respect to the previous position of observer 2*. This is the significance of  $dr$  in the comoving metric (1). And since in this sense,  $dr \neq 0$ ,  $V \neq 0$  for a dynamic fluid:

$$V = \frac{\sqrt{-g_{rr}}dr}{\sqrt{g_{tt}}dt} \quad (16)$$

Note that in the foregoing Eq.  $dr$  is the distance between two neighbouring layers as recorded in the first snapshot, i.e,  $dr$  is measured at a fixed  $t$ . And in Eq.(16),  $dt$  is measured solely by the 1st clock, i.e, by a clock fixed at  $r = r$ . Therefore, the  $dr/dt$  term appearing in Eq.(16) can actually be written as

$$V = \frac{\sqrt{-g_{rr}}dr|_t}{\sqrt{g_{tt}}dt|_r} \quad (17)$$

Also note that since

$$\frac{\partial R}{\partial r} = \frac{dR}{dr}|_t \quad (18)$$

and

$$\frac{\partial R}{\partial t} = \frac{dR}{dt}|_r, \quad (19)$$

the  $dr/dt$  appearing in Eq.(17) may be expressed as

$$\frac{dr}{dt} = \frac{(\partial R/\partial t)}{(\partial R/\partial r)} \quad (20)$$

Now by using Eqs.(3), (4) and (20), we find that Eq.(16) can be further rewritten as  $V = U/\Gamma$ . In this way we have a complete explanation as to how Eq.(5) is valid even though  $\Gamma$  and  $U$  involve partial derivatives.

However speed of a given fluid element measured by an observer 1 comoving with the same fluid element is of course zero *unless* the measurement is done with respect to the adjacent observer 2,  $V_{com} \equiv 0$ . This is true in Newtonian hydrodynamics too. So one must distinguish between these two notions of comoving speed. When this latter view is adopted the comoving metric will be

$$ds^2 = g_{tt}dt^2 \quad (21)$$

rather than Eq. (1); and corresponding  $V_{com} \equiv 0$ . We may recall that in the concept of the 4 velocity of the fluid in the comoving frame  $u^\mu$ , one tracks a given fluid element irrespective of its relative displacement from a neighbouring element and hence the spatial part of  $u^\mu$  involves  $V_{com} = 0$  rather than  $V$ . While we explain the physical meaning of  $V$  we remind the reader again that in the derivation of Eqs. (2) and (14) *we did not even require to know what the nomenclature  $V$  stood for*, neither did we require to impose the condition that  $V \leq 1$ . For previous use of the concept of speed in the context of comoving coordinates see Eqs. 2.79-80 in ref. [21] and Eqs. 6.42-45 in ref. [22].

### III. COMPLETELY INDEPENDENT PROOFS

It is likely that some readers would just refuse to accept the derivation which led to Eq. (2) even if he/she would not be able to point out any specific error in it. Let us remind them that, our basic result that there cannot be any horizon at a finite value of  $R$  can be obtained from several other independent considerations too:

#### A. Radial Geodesic Around a Schwarzschild BH

Let us assume, for the time being, the existence of a Schwarzschild BH with finite mass  $M$  and horizon size  $R_g = 2M$ . The spacetime for  $R \geq R_g$  is described by

$$ds^2 = dT^2(1 - 2M/R) - \frac{dR^2}{(1 - 2M/R)} - R^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (22)$$

For radial geodesics of test particles,  $d\theta = d\phi = 0$ , and it follows that along the radial geodesic [23]

$$\left(\frac{dR}{dT}\right)^2 = \frac{(1 - 2M/R)^2}{E^2} [E^2 - (1 - 2M/R)] \quad (23)$$

where  $E$  is the conserved energy per unit rest mass. As  $R \rightarrow 2M$ , the foregoing Eq. shows that

$$\left(\frac{dR}{dT}\right)^2 \rightarrow (1 - 2M/R)^2 \quad (24)$$

By transposing this Eq., we find that

$$dT^2(1 - 2M/R) \rightarrow \frac{dR^2}{1 - 2M/R} = dz^2 \quad (\text{say}) \quad (25)$$

By inserting the above relation in Eq.(22), we find that, for a radial geodesic, as  $R \rightarrow 2M$

$$ds^2 \rightarrow dz^2 - dz^2 \rightarrow 0 \quad (26)$$

Note that this is an *exact and straightforward result* and nothing can be done to undo it.

This means that, the *timelike geodesic* would turn null *if an EH would exist*. But this is not possible if EH is really a regular region of spacetime and it demands that there cannot be any finite mass Sch. BH with a finite horizon ( $R_g = 2M > 0$ ). But if we still insist on having a BH, the only honourable solution here would be to recognize that the EH is actually a true singularity, i.e, the central singularity. This would mean that  $R_g = 0$  or  $M = 0$ . So if we

assume that there is a BH, the only admissible value of its mass is  $M = 0$  or else we may have finite mass objects which could be very similar to BHs in many ways, but which are not exactly BHs. Also note that from Eq.(24), eventually, at  $R = 2M$  we have

$$\frac{dR}{dT} \rightarrow 0; \quad dR \rightarrow 0 \quad (27)$$

because by definition  $dT$  is an infinitesimal. At this point, some of the readers may have one confusion: It is well known that  $\Delta T_g = \int_R^{R_g} dT = \infty$ , and some readers may confuse  $\Delta T_g$  with  $dT$  and may erroneously think that  $dT = \infty$ . The fact that  $dR = 0$  at  $R = 2M$  indicates that the EH is the true end of the spacetime or it is the central singularity  $R = 0$ .

### B. Same Result Using Kruskal Coordinates

If  $u$  and  $v$  are the Kruskal coordinates the corresponding metric is [1,23]

$$ds^2 = \frac{32M^3}{R} e^{-R/2M} (dv^2 - du^2) \quad (28)$$

and over the entire Kruskal diagram, we have [23]

$$u^2 - v^2 = \left( \frac{R}{2M} - 1 \right) e^{R/2M} \quad (29)$$

so that

$$u^2 = v^2; \quad v/u = \pm 1, \quad R = 2M \quad (30)$$

By differentiating Eq.(29) by  $T$ , we find

$$2u \frac{du}{dT} - 2v \frac{dv}{dT} = \frac{e^{R/2M}}{2M} \frac{R}{2M} \frac{dR}{dT} \quad (31)$$

Using Eq.(27) in the foregoing Eq., we find that the R.H.S. of Eq.(31) is zero at  $R = 2M$  and

$$\frac{du}{dv} \rightarrow \frac{v}{u}; \quad R \rightarrow 2M \quad (32)$$

Now combining Eqs.(30) and (32), we obtain

$$(du/dv) \rightarrow \pm 1; \quad du^2 \rightarrow dv^2, \quad R \rightarrow 2M \quad (33)$$

Hence the Kruskal metric

$$ds^2 = \frac{32M^3}{R} e^{-R/2M} (dv^2 - du^2) \rightarrow 0; \quad R \rightarrow 2M \quad (34)$$

Again note that this is an absolutely straight forward and exact result and nothing can be done to undo it. Yet some of the readers may develop a confusion here and on the basis of which they might feel that the Eq.(34) could be incorrect because of the following reason: If we divide both side of Eq.(28) by  $ds^2$  we would obtain a first integral of motion along a radial geodesic:

$$\frac{32M^3}{R} e^{-R/2M} [(dv/ds)^2 - (du/ds)^2] = 1 \quad (35)$$

anywhere including at  $R = 2M$ . Eq.(35) may convey an impression that  $dv^2 > du^2$  everywhere. But it would be incorrect to assume so in general. All that this identity shows is that

$$\frac{dv^2 - du^2}{ds^2} > 0 \quad (36)$$

If there is a fraction  $z = y/x > 0$ , it does not necessarily mean  $y > 0, x > 0$  because a  $0/0$  form can also be  $> 0$ . Seen more clearly, Eq.(35) is nothing but the following *identity*:

$$\frac{ds^2}{ds^2} = 1 \quad (37)$$

And an identity like  $x/x = 1$  cannot determine the value of  $x$  by itself, all it means is that  $x = x$  where  $x$  can be both  $> 0, < 0$  or  $= 0$ .

There might yet another illogical attempt to deny the correctness of Eqs. (26) and (34) in the following manner. Some readers may insist that Eq. (23) is not extendible upto  $R = 2M$  at all because the  $R, T$  coordinates “break down” there. But if the “ $R, T$ ” coordinate system would really break down, the Kruskal coordinates should also “break down” because *Kruskal coordinates are not directly linked to any invariant, any scalar or any observables, but on the other hand, they are built using solely  $R$  and  $T$* . On the other hand,  $R$  is directly related to *invariant* circumference radius/ $2\pi$ . Further, since the curvarure *scalar* is  $K = 48M^2/R^6$ ,  $R$  is also inversely proportional to the 1/6 th power of  $K$ . Similarly,  $T$  appearing here is the *proper time* measured by a distant observer  $S_\infty$  and thus  $T$  too has a solid physical significance unlike  $u$  and  $v$ . So if the EH or the region interior to it are to be described by  $u = u(R, T)$  and  $v = v(R, T)$ , by definition,  $R$  and  $T$  must remain valid coordinates there. Thus Eq. (23) must remain valid wherever  $u$  and  $v$  are definable even though Eq. (22) may not be valid.

Had we differentiated Eq.(29) by  $v$  or  $R$  instead of  $T$  we would have obtained the same result because it can be seen that

$$\frac{dv}{dR} = \frac{rv}{8M^2}(R/2M - 1)^{-1} + \frac{u}{4M} \frac{dT}{dR} = \infty; \quad R \rightarrow 2M \quad (38)$$

GTR demands that the geodesic for a material particle must be timelike,  $ds^2 > 0$  at any regular non-singular region of spacetime. But we found that, on the horizon, we would have  $ds^2 = 0$  along the geodesic. Since this would be a violation of GTR it means that the assumption of existence of a finite mass BH and its non-singular horizon ( $R_g = 2M > 0$ ) is incorrect. Mathematically, if a BH would exist, its mass must be  $M = 0$  so that the EH,  $R_g = 2M = 0$ , is a true physical singularity and not a mere coordinate singularity.

Also the occurrence of  $ds^2 = 0$  *along a radial geodesic* must not be confused with the fact that the EH is a null hypersurface. In the 1D Minkowski spacetime diagram, the  $45^\circ$  lines constitute the null surface and it does not automatically mean that a timelike geodesic approaching these null lines would turn null.

So far, as noted by the present author, the only scientific critique to our above result is due to Tereno [24–27] who insisted that  $|du/dv| < 1$  ar  $R = 2M$  (Tereno like us also justifiably used Eq.(23) over the entire spacetime). But in Eq. (33), we have already found that  $du/dv$  is indeed  $\pm 1$  at  $R = 2M$ . Then why did Tereno apparently obtain a differnt result? This is because Tereno chose not the simpler way of using Eq.(29) to obtain  $du/dv$ . On the other hand, he chose to find  $du/dv$  directly [1,24,25]:

$$\frac{du}{dv} = \frac{u - vE/\sqrt{E^2 - 1 + 2M/R}}{v - uE/\sqrt{E^2 - 1 + 2M/R}} \quad (39)$$

While on the EH,  $u_H = \pm v_H$ , the term  $x = \sqrt{E^2 - 1 + 2M/R} = \pm E(1 - \epsilon/2E^2)$  close to the EH with  $\epsilon = 1 - 2M/R \rightarrow 0$ . Eq.(39) thus can assume 4 set of values:

Case I:  $x = +E(1 - \epsilon/2E^2)$ ;  $u_H = -v_H$ ,  $du/dv = -1$

Case II:  $x = +E(1 - \epsilon/2E^2)$ ;  $u_H = +v_H$ ,

$$\frac{du}{dv} = \frac{(u_H/E^2) - (e/u_H)}{(u_H/E^2) + (e/u_H)} \quad (40)$$

Case III:  $x = +E(1 - \epsilon/2E^2)$ ;  $u_H = +v_H$ ,  $du/dv = +1$

Case IV:  $x = +E(1 - \epsilon/2E^2)$ ;  $u_H = -v_H$  and

$$\frac{du}{dv} = \frac{(e/u_H) - (u_H/E^2)}{(e/u_H) + (u_H/E^2)} \quad (41)$$

While Cases I and III do show that  $|du/dv| = 1$  as clearly seen before, Cases II and IV may lend an impression that, at the same time,  $|du/dv|$  could be  $< 1$ . If this would happen,  $ds^2$  would be both 0 and  $> 0$  ar  $R = 2M$ . But there must be a unique value and sign of  $ds^2$ , and therefore, we must have  $u_H^2 = v_H^2$  equal to either 0 or  $\infty$ .

To obtain the explicit form of  $u$  and  $v$  for a radial geodesic in terms of only  $R$ , we need to recall the relationship between  $T$  and  $R$  which, in turn, is obtained by integrating Eq.(23) [23]:

$$\frac{T}{2M} = \ln \frac{(R_i/2M - 1)^{1/2} + \tan(\eta/2)}{(R_i/2M - 1)^{1/2} - \tan(\eta/2)} + \left(\frac{R_i}{2M} - 1\right)^{1/2} \left[ \eta + \left(\frac{R_i}{4M}\right) (\eta + \sin \eta) \right] \quad (42)$$

Here the test particle is assumed to be at rest at  $R = R_i$  at  $T = 0$  (or for dust collapse, the starting point) and the ‘‘cyclic coordinate’’  $\eta$  is defined by

$$R = \frac{R_i}{2}(1 + \cos \eta) \quad (43)$$

The initial value of  $R = R_i = 2M/(1 - E^2)$ . We find from Eq.(42) that, as  $R \rightarrow 2M$ , the logarithmic term blows up and  $T \rightarrow \infty$ , which is a well known result. Correspondingly, as one enters the EH, one would have

$$\cosh \frac{T}{4M} \rightarrow \sinh \frac{T}{4M} \rightarrow \frac{e^{T/4M}}{2} = \infty \quad (44)$$

Then it can be found that

$$u_H^2 = v_H^2 = 4e(1 - 2M/R_i) \exp \left\{ \left( \frac{R_i}{2M} - 1 \right)^{1/2} [\eta_H + (R_i/2M)(\eta_H + \sin \eta_H)] \right\} \quad (45)$$

where

$$\eta_H = \arccos(4M/R_i - 1) \quad (46)$$

Since  $R_i > 2M$ ,  $u_H^2 > 0$ . And in terms of  $E$ , we have

$$u_H^2 = v_H^2 = 4eE^2 \exp \left( \eta_H + \frac{\eta_H + \sin \eta_H}{2\sqrt{1 - E^2}} \right) \quad (47)$$

This shows that for  $E = 1$  ( $R_i = \infty$ ),  $u_H^2 = v_H^2 = \infty$  for any pre-assumed value of  $M$  as was concluded above. But are  $u_H$  and  $v_H$  finite for  $E < 1$ ? If so, again, we would have both  $ds^2 = 0$  and  $\neq 0$  at  $R = 2M$ . But since this is not possible, we must have indeed have  $u_H = v_H = \infty$ , which, by virtue of Eq.(45) means that, for consistency, we must have  $M = 0$ .

### C. Physical Interpretation

Both the Sch. metric (22) and Kruskal metric (28) can also yield an Eq. like that of (10) and from which, we would see that, occurrence of  $ds^2 = 0$  on the EH physically means occurrence of either  $g_{00} = 0$  or  $V = 1$  or both of them. And it is already well known that in Sch. coordinate the local 3 speed of a free particle becomes equal to speed of light at  $R = 2M$  because [1,23]

$$V_{sc}^2 = 1 - \frac{(1 - 2M/R)}{E^2} = 1; \quad R = 2M \quad (48)$$

As mentioned by Landau & Lifshitz [15] any arbitrary metric associated with the gravitational field of single body can be written as

$$ds^2 = g_{00}dx^{02}(1 - V^2) \quad (49)$$

where  $V$  is appropriately defined 3-speed. Therefore, Eq.(48) itself could have been taken as a proof that  $ds^2(Sch) = 0$  at EH (along a radial geodesic) because  $g_{00} = g_{TT} \neq \infty$  at  $R = 2M$  (on the other hand  $g_{TT} = 0$  too at  $R = 2M$ ) and of course  $dx^0$  being an infinitesimal.



In GTR, like in Sp. Theory of Relativity, the velocity addition law is such that once any observer measures a particular speed  $V = 1$ , *all other observers (coordinates) too measure the same speed* and this is the reason that all observers measure speed of light  $= c = 1$  in GTR too. Thus the speed of the test particle at the EH as measured by the Kruskal coordinates must also be  $V_{Kr} = 1$ . The 3-speed in terms of Kruskal coordinates is

$$V_{Kr} \equiv \frac{dl}{d\tau} = \frac{\sqrt{-g_{uu}}du}{\sqrt{g_{vv}}dv} = \frac{du}{dv} \quad (50)$$

because in Eq.(28),  $g_{vv} = -g_{uu}$ . By using Eq.(33), we find that indeed  $V_{Kr} = 1$  at  $R = 2M$ . Further since  $ds^2$  is an invariant, we must have  $ds^2(Kruskal) = 0$  (at  $R = 2M$ ) once  $ds^2(Sch) = 0$  (at  $R = 2M$ ). Thus Eqs.(26) and (34) are fully self consistent. Hence there is a complete physical as well as mathematical consistency between subsections A, B and C. Such an allround consistency would not have been possible had any of the results contained in these subsections been incorrect. In fact, now *Tereno too has admitted* that, the local speed as defined by all the authors becomes  $c$  at  $R = 2M$  irrespective of the coordinate system used [28]; (he has not cited his own previous preprints). He has also correctly admitted that, therefore, one should have  $V > c$  inside the EH [28,29]. However, to avoid such a prospect, Tereno has appealed that the basic concept of local speed as defined by Landau and all other authors be changed [28]!! It can be shown that as per his prescription, all local speeds of free falling particles should be zero because they are measured by another free falling observer situated at the same instantaneous location (but the assumption of existence of a BH must be retained)!

This section shows that for a *point mass* the mathematical solution (22) becomes physically meaningful too only if  $M = 0$ . This means that a BH of mass  $M = 0$  could be the the endpoint of gravitational collapse. Then this is a hint that in our constraint (2), physically, we should have  $2M/R = 1$  rather than  $2M/R < 1$ .

#### D. Same Constraint From OS Paper

The Eq.(36) of Oppenheimer Snyder (OS) paper [5] looks like

$$T \sim \ln \frac{y^{1/2} - 1}{y^{1/2} + 1} + \text{other terms} \quad (51)$$

where  $y = R/2M$  on the boundary of the collapsing star. In order that the argument of the log term in the above Eq. is positive and  $T$ , the *proper time* of a distant observer, is definable, it is necessary that  $y \geq 1$  or  $2M/R \leq 1$ . Thus the condition (2) that there is no trapped surface is very much steeped into Eq.(51). Therefore final mass of the dust ball  $M_f = 0$  at  $R = 0$ . But if the “dust” is (incorrectly) treated as a *coherent continuous physical fluid* it would not be able to radiate anything including gravitational radiation (because of assumed isotropy). In such a case the mass of the dust ball must remain constant and hence even the initial mass of it  $M_i = 0$ ! This means that if we (incorrectly) insist that a spherical “dust” is treated as a *coherent continuous physical fluid* the only allowed value of its mass is  $M = 0$  and particle number  $N = 0$ . Physically this means that if we strictly demand that a fluid has no pressure at all  $p \equiv 0$ , it cannot really be considered as a coherent continuous physical fluid.

Thus a finite mass spherical “dust” with  $p \equiv 0$  is merely a collection of *incoherent non-continuous particles*. In such a case there would be free space between the “dust” particles and *even though the system is spherically symmetric, the system is not isotropic*. Thus a spherical “dust” of  $N$  particles is an assembly of  $N/2$  *incoherent* pairs of particles. In GTR any mutually attracting pair of particles emit gravitational radiation unmindful of the presence of other incoherent pairs. Thus actually the gravitational mass of a true incoherent non-continuous “dust” would vary during the process of collapse like that of a physical fluid. And hence it is possible that as  $R$  decreases,  $2M/R \leq 1$  during its evolution. So whereas a spherical physical fluid (isotropic) would decrease its mass by emission of electromagnetic or neutrino radiation a spherical (yet non-isotropic) “dust” would decrease its mass by emission of gravitational radiation. But in either case  $2M/R \leq 1$ .

Some readers may argue that (i) the work of OS, in any case, has been redone in various form by various other authors and which show that collapse of homogeneous dust results in a finite mass BH or, (ii) there should have been a modular sign in Eq.(36) of OS (which they may have forgot to put!), and, when such a sign is introduced in Eq.(51), there would be no problem in allowing a value of  $y < 1$  or in having  $2M/R > 1$ .

When one neglects the emission of (gravitational) radiation from incoherent dust, its mass  $M$  remains fixed, and if one starts with a finite mass  $M$  and ignores the inherent implication of Eq.(36) of OS (and also ignore other inconsistencies discussed in Paper I), one can happily conclude that trapped surfaces are formed and  $2M/R > 1$  as  $R$

becomes sufficiently small. The physical inconsistency in assuming the initial condition of  $M = \text{finite}$  would however always be present through the modified  $T - R$  relationship in any such study.

In the absence of any pressure and neglect of radiation, a given dust particle undergoes free fall and behaves like a free test particle (note that actually a pair of mutually accelerating particles do emit gravitational radiation and this is ignored in this “free particle” scenario). In any case, once one assumes this “free particle” view point, the physics of dust collapse is somewhat similar to the radial inward motion of a “free particle” around a BH of mass  $M$ . The relation between  $T$  and  $R$  is same as before and is given by Eq.(42).

Since  $\tan(\eta/2) = (R_i/R - 1)^{1/2}$ , we may rewrite Eq. (42) in terms of a new variable

$$x = \left( \frac{R_i/2M - 1}{R_i/R - 1} \right)^{1/2} \quad (52)$$

as

$$\frac{T}{2M} = \ln \frac{x+1}{x-1} + \left( \frac{R_i}{2M} - 1 \right) \left[ \eta + \left( \frac{R_i}{4M} \right) (\eta + \sin \eta) \right] \quad (53)$$

It can be easily found that, irrespective of  $M$  being finite or zero,  $T \rightarrow \infty$  as  $x \rightarrow 1$  or  $R \rightarrow 2M$ . For the time being, we assume that  $M \geq 0$ . Then, from Eq.(52), note that

$$x \leq 1; \quad \text{for } R \leq 2M \quad (54)$$

Thus for  $R < 2M$ ,  $T$  is *not definable at all* because the argument of logarithmic function in Eq.(53) can not be negative. This is similar to the problem encountered in Eq.(51). However nobody seems to have pondered *whether the situation which is leading to an imaginary  $T$  is unphysical or not*. As mentioned before,  $T$  is the *proper time* measured by a distant observer  $S_\infty$  and thus  $T$  cannot be imaginary. Note, as far as the distant inertial observer  $S_\infty$  is concerned, he is either able to watch an event ( $T = \text{finite}$ ) or unable to do so ( $T = \infty$ ). Probably later authors realized that  $T$  can not be allowed to be imaginary, not because of the fact that  $T$  is still the *proper time* of a Galilean observer, a measurable quantity, but because of the fact that, otherwise the esoteric new coordinates, like  $u$  and  $v$  would not be definable. Thus we find that a modulus sign was introduced in the  $T - R$  relationship [23]:

$$\frac{T}{2M} = \ln \left| \frac{x+1}{x-1} \right| + \left( \frac{R_i}{2M} - 1 \right) \left[ \eta + \left( \frac{R_i}{4M} \right) (\eta + \sin \eta) \right] \quad (55)$$

But, the conceptual catastrophe actually becomes worse by this tailoring. Of course, as the particle enters the EH, still,  $T \rightarrow \infty$ . Note that,  $T$  having become infinite, definitely can not decrease, and more importantly **inside the EH,  $T$  can not be finite** because, otherwise, *it would appear that although the distant observer can not witness the exact formation of the EH (in a finite time), nevertheless, he can witness the motion of the test particle inside the EH*. This would mean violation of causality and the existence of some sort of a “time machine”. And we know that, it is only the comoving observer who is supposed to witness both the formation of EH and the collapse beyond it. But the foregoing equation tells that if  $M > 0$  (finite), the *the value of  $T$  not only starts decreasing* but also suddenly **becomes finite** as the boundary enters the EH ( $R < 2M$ )!! And as the collapse is complete, i.e.,

$$x = 0; \quad \text{if } M > 0; \quad \text{at } R = 0 \quad (56)$$

the corresponding value of  $T = T_0$  required by the distant observer to see the **collapse to the central singularity within the EH** is simply

$$T = T_0 = 2M \left( \frac{R_i}{2M} - 1 \right) \left[ \eta + \left( \frac{R_i}{4M} \right) (\eta + \sin \eta) \right] \quad (57)$$

By using Eq.(43), this can be rewritten as

$$T_0 = \pi(R_i - 2M) \left( 1 + \frac{R_i}{4M} \right) \quad (58)$$

And note that, if we really insist that  $T_0 = \infty$  as per the original agenda, i.e, if we really insist that events occurring within the EH are not visible to the distant observer, we must realize that  $M = 0$ ! This would mean (i) there is no additional spacetime between the EH and the central singularity, i.e, they are synonymous and the Sch. singularity is a genuine singularity provided one realizes that by the time one would have  $R = 2M$ , the value of  $M \rightarrow 0$ , and (ii) the proper comoving time for formation of this singularity is  $\tau \propto M^{-1/2} = \infty$ . The latter means that, it is not formed at any finite proper time, the collapse process continues indefinitely and there is *no incompleteness for the timelike geodesics*.

There is yet another argument which corroborates the result obtained in Paper I. It is known that the radial component of acceleration ( $a^R$ , a coordinate dependent quantity) acting on a test particle around a Sch. BH blows up at  $R = 2M$ . Since  $a^R$  itself is a physically measurable quantity, its blowing up (eventhough it is coordinate dependent) should have initiated introspection on the reality of occurrence of the horizon. But conveniently this physical question is avoided with the hyperbole of “coordinate singularity”. For the sake of argument, let us too accept it for a moment. But we can construct an acceleration INVARIANT/SCALAR (a coordinate independent quantity) [30–33]

$$a = \sqrt{a^i a_i} = \frac{M}{R^2 \sqrt{1 - 2M/R}} \quad (59)$$

Note that *this scalar blows up* at  $R = 2M$ . Further, had there been a spacetime beneath  $R < 2M$ , it would have become imaginary there. Since  $a$  is a coordinate independent quantity now *we cannot get away by blaming it on coordinate singularity*.

The only solution lies here is the admission of the fact that  $M = 0$ , the horizon is at  $R_g = 2M = 0$  (the central singularity) and there is no spacetime beneath the horizon.

#### IV. SCHWARZSCHILD OR HILBERT SOLUTION

As they say, sometime truth is stranger than fiction: It has been pointed out recently that the conventional Sch. solution is not due to Schwarzschild at all, on the other hand, it is due to Hilbert. The original paper of Schwarzschild [34] where he worked out the spacetime around a *point mass*,  $M$  has recently been translated into English by Loinger and Antoci [31]. Even before this Abrams [30] pointed out that, the original Schwarzschild solution is

$$ds^2 = dT^2 \left( 1 - \frac{2M}{R} \right) - \frac{dR_*^2}{\left( 1 - \frac{2M}{R} \right)} - R^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (60)$$

where

$$R = [R_*^3 + (2M)^3]^{1/3} \quad (61)$$

The important point is that here  $R$  is not the radial variable, on the other hand the radial variable is  $R_*$ ; the *point mass is sitting at  $R_* = 0$  and not at  $R = 0$* . Thus at,  $R_* = 2M$ , obviously there is no singularity, whereas there is a central singularity at  $R_* = 0$ . Since there is no spacetime beneath  $R_* < 0$ , there cannot be any spacetime beneath  $R < 2M$ . The prevalent Sch. solution, which is actually the Hilbert solution is, however, still a mathematically valid solution for a “point mass”. So the question as to which is the physically valid solution, has to be decided on physical reasoning. Here, some authors [30,31,35–37] have rejected the Hilbert solution because, according to it, there would be a spacetime beneath  $R < 2M$  and the scalar  $a$  would then first blow up and then become imaginary. On this ground, these authors, claim that the original Schwarzschild solution is the physically valid solution. If so, in this picture, obviously, there would be no BH. On the other hand, there is a problem with the original Sch. solution; as admitted by Sch. himself [34], in the limit of weak gravity, his equation does not exactly reduce to a Newtonian form. On the other hand, we know that, the prevalent Sch. solution, i.e, the Hilbert solution does yield the correct Newtonian form. Further, the angular part of either Schwarzschild or Hilbert solution (or any spherically symmetric metric) shows that  $R$  rather than  $R_*$  is the Invariant Circumference Coordinate. Therefore, Hilbert solution cannot really be rejected. Then how do we resolve this paradox? We note that, the problem lies with our premises that “there is a *point mass* with a finite gravitational mass”! And the solution lies in realizing that if there is a body of finite mass, it cannot be considered as a *geometrical point* even at a classical level. On the other hand, if we insist that there is a point mass, its gravitational mass must be zero. When  $M = 0$ , Sch. solution and Hilbert soln. become identical,  $R_* \equiv R$ , and there is no EH, but there is only a central singularity in a mathematical sense. One may note here that in classical electrodynamics there are “point charges”. And existence of “point charges” implies singularities. On the other hand, in quantum field theories, there are no point charges, for instance, an electron is actually not a “point charge”. We find here that in GTR, even at a classical, non-quantum level, there cannot be any “point mass” (having finite  $M$ ).

Having confirmed the basic result obtained in Paper I by several independent modes, now we shall focus attention on the salient features of the final states of spherical gravitational collapse as dictated by Eqs.(5) and (8). Before we do this, see that all the initial states may be considered to have  $U_i = V_i = 0$  and  $\Gamma_i^2 = 1 - 2M_i/R_i$ .

We have found that, as to the final state, the master Eqs.(5) and (8) admit basically four types of solution:

### A. Dust Solution

We discussed in Paper I that if a dust is (incorrectly) considered as a coherent continuous physical fluid, for the dust solutions,  $\Gamma_f$  remains pegged to its initial value  $\Gamma_i$ . Physically, this is so because, a such a (incorrect) dust solution is necessarily radiationless, and comparable to free fall of a test particle, for which  $\Gamma = \Gamma_0 =$  *conserved energy per unit rest mass*. And there is no question of  $\Gamma$  assuming a negative value in such a case: In this case the final state could be marked with

$$\Gamma_f^2 = U_f^2 > 0, \quad V_f^2 = 1, \quad 2M_f/R_f = 1. \quad (62)$$

In Paper I, we have seen that Oppenheimer-Volkoff (OV) equation [38] demands that for such dust solutions  $\rho \equiv 0$ , and consequently, not only  $M_f = 0$ , but also  $M_i = 0$ . This is possible in a strict sense only if  $N \equiv 0$ . Thus we shall not consider this case any further.

### B. Static Final States With $2M/R < 1$

Static final states will be marked by  $V_f = U_f = 0$  with  $\Gamma_f > 0$ :

$$\Gamma_f^2 > 0, \quad U_f = V_f = 0, \quad 2M_f/R_f < 1 \quad (63)$$

### C. Static Final State With $2M/R = 1$

$$\Gamma_f^2 = U_f^2 = 0, \quad V_f = 0, \quad 2M_f/R_f = 1 \quad (64)$$

### D. Dynamic Final State With $2M/R = 1$

A dynamic final state will have  $V_f > 0$ . But since  $V_f$  is allowed to be non-zero in this case, the *final state* must be marked with the maximum possible value of  $V_f = 1$ :

$$\Gamma_f^2 = U_f^2 = 0, \quad V_f^2 = 1, \quad 2M_f/R_f = 1. \quad (65)$$

## VI. MORE ON THE PHYSICAL SOLUTIONS

We may note that all the physical solutions correspond to  $U_f \rightarrow 0$ . A physically meaningful final state must have  $U_f = 0$  because of the simple fact that the final state, by definition, must correspond to an extremum of  $R$  and therefore, we must have  $U_f = dR/d\tau = 0$ . However, it does not mean that the march towards the final state would be monotonically decelerated for a real fluid. Depending on the unpredictable actual solutions of the complicated non-linear coupled collapse equations and the radiation transfer mechanism, the system may even undergo phases of acceleration and deceleration in the fashion of a damped oscillator. In contrast, a conventional (incorrect) dust-collapse picture would suggest monotonic acceleration and no emission of radiation. In such a picture, a neutron star (NS) would be born on a free fall time scale  $< 1$  ms. But we know that actual microphysics intervenes in such a case and ensures that the  $\nu$  signal heralding the birth of the NS is dictated by radiation diffusion time scale, which, in this case is  $\sim 10$ s.

And since it is  $U$  and not  $V$  which appears in the collapse equations, in a sense, the final state here corresponds to the static Oppenheimer-Volkoff limit:

$$\partial R/\partial r \rightarrow 1; \quad R \rightarrow 0, \quad (66)$$

$$-g_{rr}^{1/2} = e^\lambda = \frac{1}{\Gamma} \frac{\partial R}{\partial r} \rightarrow \frac{1}{\Gamma}, \quad (67)$$

$$\frac{dM}{dR} \rightarrow 4\pi R^2 \rho; \quad R \rightarrow 0, \quad (68)$$

and

$$\frac{dp}{dR} \rightarrow -\frac{(\rho + p)(4\pi p R^3 + M)}{\Gamma R^2}. \quad (69)$$

Note, however, that the  $U = 0$  case truly corresponds to a *locally static* case only as long as  $V_f = 0$ , such as cases V.B and C.

In general, we see that that  $\Gamma_f \geq 0$ , and  $\Gamma_f$  cannot assume any negative value. This conclusion could have been directly drawn from Eq.(5) as well. If the fluid is collapsing (expanding), on physical grounds both  $U$  and  $V$  must be negative (positive), implying  $\Gamma \geq 0$ . Also by using the general condition that  $M_f \geq 0$ , it can be shown that  $\Gamma_f \geq 0$ .

#### A. Final State With $2M/R < 1$

The case IV.B corresponding to  $2GM_f/R_f < 1$  with  $V_f = 0$  indicates the formation of static Ultra Compact Objects (UCOs) of finite size  $R_f$ . An UCO may be considered as a special case of ECO with  $V_f = 0$ . We shall show shortly that if one insists for a  $R_f = 0$  solution in this, the configuration would have zero baryon number. Thus this solution indeed corresponds to finite size static UCOs.

To some extent, the solutions for this case have already been discussed in the literature [38,39]. OV used ideal Fermi-Dirac EOS and sought solutions with central density  $\rho_c = \infty$  and central pressure  $p_c = \infty$ :

$$\rho \rightarrow \frac{3}{56\pi R^2}, \quad (70)$$

$$p \rightarrow \frac{1}{3}\rho, \quad (71)$$

$$M \rightarrow \frac{3}{14G}R, \quad (72)$$

$$\Gamma = \text{constant} = (4/7)^{1/2} \quad (73)$$

Here it should be remembered that, as long as  $R(t) \neq 0$ , i.e., when one is dealing with a non-singular configuration which may nevertheless harbour a singularity at the center, the usual boundary condition that (i)  $p = 0$ , at  $R = R_0$ , the external surface, must be honoured. And only when a complete singular state is reached with  $R_0 = 0$ , the solution must be allowed to be truly discontinuous in  $p$  and  $\rho$ . This shows that a strictly correct solution of the above referred OV solution could be of two types: (1)  $R_0 = \infty$ ,  $M(R_0) = \infty$ , which is an unstable solution except for photons; (2)  $R_0 = R = 0$ ,  $M(R_0) = 0$ , and this the only stable solution for baryons. This conclusion also follows from the solution of Misner & Zepolsky [39] which shows that  $M_{\text{core}} = 0$ ,  $R_{\text{core}} = 0$  when  $\rho_c$  is indeed  $\infty$ . And, though, the latter OV solution is a stable one, it is devoid of any physical content because, as was specifically discussed by OV, it has  $N = 0$ . This can be verified in the following way. Note that since luminosity  $L \rightarrow 0$  in the static limit, we must have  $p_{\text{radiation}} \rightarrow 0$  as  $R \rightarrow 0$ , so that the total pressure  $p = p_{\text{matter}}$  in this limit. In the low density limit degenerate fermions have an EOS  $p \propto n^{5/3}/m$ . Since this EOS depends on the value of  $m$ , it is possible that in the low density limit, it depends on the specific nature/variety of the fermions. In contrast, as  $\rho \rightarrow \infty$ , the fermi EOS becomes independent of  $m$

$$p = 0.25\sqrt{3/8\pi}hcn^{4/3} \quad (74)$$

where  $h$  is Planck's constant. In fact, it may be shown that black body photons tend to obey the  $p \propto n^{4/3}$  EOS and it is likely that all extremely degenerate bosons with a finite value of  $m$  tends to follow this EOS in the limit  $\rho/m \rightarrow \infty$ . In any case for the fermions, Eqs.(70), (71), and (74) would suggest that

$$n = n_0 R^{-3/2} \quad (75)$$

where  $n_0$  is an appropriate constant. Note that for static solutions, by using Eqs.(67) and (72), we have

$$N = \int_0^{R_0} 4\pi n e^\lambda R^2 dr = 4\pi n_0 \int_0^{R_0} \frac{R^{1/2}}{\Gamma} dR \quad (76)$$

From this Eq. it is seen that for a constant  $\Gamma \neq 0$ ,  $N \rightarrow 0$  as  $R_0 \rightarrow 0$ . It can be verified that, even if we used a more general form of Eq.(74) like  $p \propto n^{\frac{4+\alpha}{3}}$ , for the  $\Gamma_f > 0$  case all singular solutions have  $N = 0$  if  $\alpha < 1$ . Therefore as far as final states with  $R_f = 0$  are concerned, we discard the  $2GM_f/R_f < 1$  case as physically meaningless. This conclusion corroborates the spirit of the Cosmic Censorship Conjecture that spherical collapse cannot give rise any naked singularity. As mentioned before many numerical/semi-analytical studies on inhomogeneous dust collapse find the absence of formation of EH in accordance with our exact result. And this absence of EH is interpreted in terms of formation of “naked singulaities”. In a strict sense occurrence of true gravitational singularity would mean that the proper radial distance measured along radial world lines  $l = \text{finite}$ . It is likely that such numerical/semi-analytical studies implicitly assume that indeed  $l = \text{finite}$  and thus use the term “singularity” in this context. But we shall see shortly that actually  $l = \infty$  if the collapse would proceed upto  $R = 0$  and therefore eventhough there is a “nakedness” (non-occurrence of EH), there would actually be no true singularity.

However we remind that if  $R_f > 0$ ,  $2M_f/R_f < 1$  is a physically valid configuration with finite  $N$  (UCO).

### B. Solutions With $2M_f/R_f = 1$ , $M_f = 0$ , $R_f = 0$

As far as true singular states are concerned, the meaningful solutions are V. C and D with  $\Gamma_f = 0$ , because, for a physical fluid, this null value of  $\Gamma_f$  implies that  $M_f = 0$  even though  $M_i > 0$ . And out of these two probable cases, the solution D is of particular astrophysical importance. Since in this case,  $V_f \rightarrow \text{constant}(= 1)$  for all fluid elements, it signifies complete kinetic ordering of the fluid and, hence, eventual zero entropy. On the other hand, solution C might correspond to a locally static ( $V_f = 0$ ) configuration where all the fluid particles form a single coherent perfectly degenerate system. In this case too, the entropy of the system would be  $\sim \ln(1) = 0$ .

With reference to the cases V.C and D, by inspection, we have found that the following class of solutions are suitable:

$$\rho \rightarrow \frac{1}{8\pi GR^2}; \quad R \rightarrow 0, \quad U \rightarrow 0, \quad (77)$$

$$p \rightarrow \frac{a_1}{8\pi GR^2}; \quad a_1 > 0, \quad (78)$$

$$\frac{2GM}{R} \rightarrow 1 - \epsilon AR^{1+2a_2}; \quad a_2 < 0.5, \quad (79)$$

where  $\epsilon \ll 1$  is a tunable constant. Here the normalization constant

$$A = R_0^{2-2a_2}, \quad (80)$$

and,

$$\Gamma \rightarrow \left(1 - \frac{2GM}{R}\right)^{1/2} \rightarrow \sqrt{\epsilon A} R^{a_2+0.5}, \quad (81)$$

so that,

$$M \rightarrow \frac{R}{2G}. \quad (82)$$

By using Eqs.(76) and (81), we find that

$$N = \frac{4\pi n_0}{\sqrt{\epsilon A}} \int_0^{R_0} R^{-a_2} dR = \frac{4\pi n_0}{\sqrt{\epsilon}(1-a_2)} \quad (83)$$

Now by tuning (decreasing) the value of  $\epsilon$  one can accommodate arbitrary number of baryons in the singular state! Recall that the for the external Vaidya metric, the dynamical collapse solution obeys the following boundary condition at the outer boundary [12,14]:  $\sqrt{-g_{00}} = e^\psi = \Gamma + U$  in order that  $\psi \rightarrow 0$  as  $R \rightarrow \infty$ , i.e, to have an asymptotically flat spacetime. Then as  $R_0 \rightarrow R \rightarrow 0$  and  $U \rightarrow 0$ , we find that

$$e^\psi = \sqrt{-g_{00}} \rightarrow 0 \quad (84)$$

This shows that the *total surface red-shift*  $z \rightarrow \infty$  *even though*  $M \rightarrow 0$ . Similarly, by using Eq.(79), we find that, the typical curvature component

$$R_{\vartheta\varphi}^{\vartheta\varphi} = \frac{2GM}{R^3} = \frac{1}{R^2} \rightarrow \infty \quad (85)$$

as  $R \rightarrow 0$ . Also, using Eq.(81), note that the proper volume of the fluid:

$$\Omega = \int_0^{R_0} \frac{4\pi R^2}{\Gamma} dR \sim \frac{1}{\sqrt{\epsilon}} R_0^{1.5} \rightarrow 0 \quad (86)$$

since  $\epsilon$  is finite. Nevertheless, most importantly, by using Eqs.(66), (67), and (81), it follows that, the proper radial distance to be travelled to reach this singular state (along a radial worldline) is

$$Lt_{R_0 \rightarrow 0} \geq \int_0^{R_0} \frac{dR}{\Gamma} \sim \frac{1}{\sqrt{\epsilon}} R_0^{0.5} \quad (87)$$

Even if  $\epsilon \neq 0$ , i.e., even if  $N$  is finite, we see that  $l \rightarrow \infty$  as  $R_0 \rightarrow 0$ ! Note that when the metric components are dependent on time, in general,  $l$  cannot be meaningfully defined [15], but here we are defining  $l$  only for a specific limit. Correspondingly, the proper time (as measured following the same radial worldline and defined in the same specific limit) required to attain this state would also be infinite

$$\tau = Lt_{R_0 \rightarrow 0} \int \frac{dl}{v} \geq \frac{l}{c} \rightarrow \infty \quad (88)$$

This means that time like radial worldlines are never terminated and *the fluid can never manage to attain this state of singularity* although it can strive to do so continuously. Now recall that, the most ideal condition for formation of a BH or any singularity is the spherically symmetric collapse of a perfect fluid not endowed with any charge, magnetic field or rotation. If no singularity can ever be formed under this most ideal condition, it is almost certain that any other realistic and more complicated case of collapse of physical matter will be free of singularities. This means that as far as dynamics of isolated bodies are concerned GTR may indeed be free of any kind of singularities (this does not rule out Big-Bang type singularities). Moreover, if a BH were formed its zero surface area would imply zero entropy content. Thus spherical gravitational collapse does not entail any loss of information, any violation of baryon/lepton number conservation (at the classical non-quantum level, i.e, excluding CP violation etc. etc.) or any other known laws of physics. The *fluid simply tries to radiate away its entire mass energy and entropy in order to seek the ultimate classical ground state*  $M = 0$ .

## VII. SUMMARY AND DISCUSSIONS

So far there have been hundreds of claims for having found observational evidences for BHs. But from several independent approaches and from most basic premises, our work has shown that GTR does not allow existence of finite mass BHs. The immediate question would be then what is the true nature of the observationally discovered BH candidates (BHC). Note that often these claims mean hint for detection of compact objects with masses larger than

the upper limit of the mass of *stable and cold* Neutron Stars (NSs) and our work too suggests existence of massive compact objects. However our work also suggests that such massive objects need not be *cold and static* and neither can they have any EH.

There are, nonetheless, claims for the “detection” of EHs in some BHCs [41,42]. The argument in favor of the claim is broadly the following: At very low accretion rates, the coupling between electrons and ions could be very weak. In such a case most of the energy of the flow lies with the ions, but since radiative efficiency of ions is very poor, a spherical flow radiates insignificant fraction of accretion energy and carries most of the energy towards the central compact object. Such a flow is called Advection Dominated Flow (ADAF). If the central object has a “hard surface”, the inflow energy is eventually radiated from the hard surface. On the other hand, if the central object is a BH, the flow energy simply disappears inside the EH. For several supermassive BHCs and stellar mass BHCs, this is claimed to be the case. But there could be several caveats in this interpretation:

(i) Garcia et al. [41] based their entire claim of having “detected” EH on a number of assumptions one of which is that X-ray binaries with similar period should have similar quiescent mass accretion rates. But we know that several compact X-ray binaries like Cygnus X-3 with orbital period  $P = 4.8$  hr and Sco X-1 ( $P=19$  hr) accrete mass at the Eddington rate. So, if we follow the argument of Garcia et al. a large number of BHC X-ray binaries with  $P$  ranging from 5.1 (GRO J0422+32) -27.0 hr (4U 1543-47) should have had near Eddington accretion rate too. But the estimated accretion rates in such cases are  $10^{-6} - 10^{-7}$  Eddington rate. Thus clearly the fundamental assumption of Garcia et al. is incorrect. The mass accretion rate, depends not only on the value of  $P$  but more importantly on factors such as the nature and evolutionary state of the mass donating companion. And in any case, for such compact binaries, if there is a viable mass transfer mechanism, the accretion rates are likely to be much higher.

(ii) Such low luminosities may not be due to accretion at all. On the other hand such low luminosities could be due to Magnetospheric emission of spinning Neutron Stars or BHCs. Recently, Robertson [3] and Robertson and Leiter [4] have *explained the X-ray emission from several BHCs having even much higher luminosities as magnetospheric origin associated with their strong intrinsic magnetic field*. Note that, if the BHCs were really BHs, they would not have had any intrinsic magnetic field whereas if they are Eternally Collapsing Objects (ECOs) or static Ultra Compact Objects (UCOs) with physical surface they are expected to have such strong intrinsic magnetic fields. Campana and Stella (2000) [43] have also argued that part of the observed emission from the BHCs could be of magnetospheric origin rather than due to accretion. Vadawale, Rao and Chakrabarti [44] have explained one additional component of hard X-rays from the micro-quasar GRS1915+105 as Synchrotron radiation.

(iii) The accretion luminosity may be radiated not only in soft X-rays but, in certain cases, also in the form of  $\gamma$ -rays or neutrinos. In other words, the luminosity in the soft X-ray band need not be proportional to the total mass accretion rate (Campana and Stella 2000) [43].

(iv) The x-rays if assumed to be of accretion origin, could be coming from an accretion disk or a corona and not from a spherical flow.

The major part of the accretion flow might also be rebounded in the form of “jets” rather than in the form of any radiation not envisaged in the original ADAF model

In a more recent review paper Narayan, Garcia and McClintock (2001) [42] have admitted that some of the assumptions behind the ADAF model may be incorrect (although on the basis of which repeated claims have been made about the “detection” of EHs in BHCs). They admit that various MHD processes (which are hardly understood) may affect the flow dynamics and the flow may be Convection dominated rather than Advection dominated. This only shows that the claims of “detection” of EH are based on poorly understood physics.

(v) Some of the BHCs in the X-ray binaries may be spinning with a period of few milliseconds. They are likely to be aspherical and potential sources of gravitational radiation. Accretion can cause such objects wobble more and radiate more gravitational radiation.

(vi) In extremely strong gravitational field associated with some BHCs ( $z \gg 0$ ), overall accretion physics may be somewhat different from what we expect from studies of NSs ( $z \sim 0.1$ ). In certain cases both accreted matter and energy might get “stuck” by extremely high gravity atleast for certain period of time and which might be reemitted later. Menou has also urged that claims of detection of “BH” cannot be made just on the basis of difference in luminosities in two classes of sources [45]. Recently Narayan and Heyl [46] have offered yet another evidence for detection of “EH”: they mention that while some of the NS X-ray binaries show Type I X-ray burst (nuclear flash on the physical boundary of the compact object) none of the BHCs show such bursts. Here they have ignored the fact that the presence of intrinsic magnetic field considerably higher than  $\sim 10^9$  G may suppress Type I burst activities in the UCOs. Thus such an absence of Type I bursts actually enhances the case that stellar mass BHCs may possess strong intrinsic magnetic field [3,4].

Therefore there is really no rigorous evidence for “detection” of any EH contrary to the repeated claims made to this effect. On the other hand, the evidence for presence of intrinsic magnetic field in the BHCs [3,4] suggests that the



BHCs are not BHs. As we show, continued collapse of sufficiently massive objects may continue indefinitely because *the ever increasing curvature of spacetime (components of Ricci Tensor) tends to stretch the physical spacetime to infinite extent*. Therefore, it is probable that, subject to the presently unknown behaviour of the arbitrary high density nuclear EOS and unknown plausible phase transitions of collapsing matter under such conditions, GTR gravitational collapse may find one or more quasi-stable ultracompact static or even dynamically collapsing configurations. Such objects, more massive and more compact than canonical Neutron Stars (NSs) have been broadly called Eternally Collapsing Objects (ECO). If in a given epoch the gravitational mass of a BHC/ECO is  $M$ , its circumference radius  $R$  can be arbitrarily close to its instantaneous Sch. radius  $R_g = 2GM/c^2$  without ever becoming less than  $R_g$ , i.e.,  $R(t) \geq R_g(t)$ . For practical purposes such objects are as compact as a supposed BH and they would satisfy many of the “operational definitions” of BHs. But whereas the EH is characterized by a gravitational redshift  $z = \infty$  such BHCs or ECOs have finite  $z$ . In case the ECO is very massive and dynamically collapsing it might try to attain the  $z = \infty$  state after infinite proper time (as measured along radial worldlines) by which time its mass would be  $M = 0$ . Thus these objects need not always be static and cold and they need not represent stable solutions of equations for hydrostatic balance. They may be collapsing with substantial local speed  $V$ , (which this work cannot predict) but the speed of collapse perceived by a distant observer ( $V_\infty$ ) would approximately be lower by a factor of  $(1+z)^2$ . Since  $V$  is finite ( $< c$ ) and eventually  $z \rightarrow \infty$ , the ultimate value of  $V_\infty \rightarrow 0$ . So it is likely that even for accurate measurements (which might be possible in remote future) spanning few years, an *isolated* ECO may appear as a “static” object. The value of  $R$  for an accreting ECO would decrease even more slowly. Thus gravitational collapse of sufficiently massive bodies should indeed result in objects which could be more compact and arbitrarily more massive than typical NSs ( $z > \sim 0.1$ ). It is found that, if there are anisotropies, in principle there could be static objects with arbitrary high (but finite)  $z$  [47]. Even within the assumption of spherical symmetry, *non-standard QCD may allow existence of cold compact objects with masses as large as  $10M_\odot$  or higher* [48].

Such stars are called Q-stars (not the usual quark stars), and they could be much more compact than a canonical NS; for instance, a stable non rotating Q-star of mass  $12M_\odot$  might have a radius of  $\sim 52$  Km. This may be compared with the value of  $R_{gb} \approx 36$  Km of a supposed BH of same mass. And, in any case, when we do away with the assumption of “cold” objects and more importantly, staticity condition there could be objects with arbitrary high  $z$ .

Recently, Mazur and Mottola [2] have suggested that when the EH would be about to be formed during collapse, the quantum back reaction due to extremely strong gravitational field would first cause a phase transition in the collapsing matter such that (i)  $p \rightarrow \rho$  and then (ii)  $p \rightarrow -\rho$ . If this would happen, the collapse would be halted and there could be static UCOs with arbitrary high mass and  $z$ . Following the theory of relativistic polytropes, we have found that even if one avoids the unusual phase transition (ii) and restricts oneself to (i) or any EOS with  $p = \beta\rho$ , with  $\beta \sim 1$ , it is possible to have UCOs of arbitrary mass but a  $z < 2$ . In particular, for stellar mass objects, if  $\beta = 1$  and such a phase transition takes place at nuclear density  $\sim 2.10^{14}$  g/cm<sup>3</sup>, the maximum mass of the UCOs would be  $\sim 11M_\odot$  which happens to be upper limit for the mass of the stellar mass BHCs [49].

Recall here that there are many globular clusters which are believed to have undergone core collapse and certainly harbour massive BHs. But all persistent observations have failed to detect the presence of any such BHs [50,51]. And as far as the active galactic nuclei are concerned, it is already a well known idea that their centers may contain supermassive stars at various stages of contraction [33,52,53] (which can very well accrete surrounding matter) or dense regions of star bursts. However, Newtonian or Post Newtonian models of supermassive stars cannot explain the sustenance of galactic nuclei for periods longer than few years. On the other hand, we have shown here that, actual Relativistic Configurations (Supermassive stars or stellar mass ECOs) may appear to be almost static for distant observers for any amount of finite duration and yet *keep on contracting internally* with finite value of  $V$ . There are some tentative evidences that the supposed supermassive BHs at the core of many galaxies may not be BHs (as confirmed by the present study):

The center of our galaxy harbours a BHC, Sgr A\*, of mass  $2.6 \times 10^6 M_\odot$ . The recent observation of  $\sim 10 - 20\%$  linear polarization from this source has strongly suggested against ADAF model [54]. On the other hand, the observed radiation is much more likely due to Synchrotron process [54]. In fact, even more recently, Donato, Ghisellini & Tagliaferri [55] have shown that the low power X-ray emission from the AGNs are due to Synchrotron process rather by accretion process.

Munyanza & Viollier [56] have claimed that the accurate studies of the motion of stars near Sgr A\* are more amenable to a scenario where it is not a BH but a self-gravitating ball of Weakly Interacting Fermions of mass  $m_f > \sim 15.9$  keV. Recall that the OV mass limit may be expressed as

$$M_{OV} = 0.54195 M_{Pl}^3 m_f^{-2} g_f^{-1/2} = 2.7821 \times 10^9 M_\odot (15 \text{ keV} / m_f)^2 (2/g_f)^2 \quad (89)$$

where  $M_{Pl} = (\hbar c/G)^{1/2}$  is the Planck mass and  $g_f$  is the degeneracy factor. With a range of  $13 < m_f < 17$  keV, these

authors point out that the entire range of supermassive BHCs can be understood. Bilic [57] has also suggested that the BHCs at the centre of galaxies could be heavy neutrino stars. Svidzinsky [58] has suggested that atleast some of the BHCs in the blazars could be heavy bosonic stars.

Note that the progenitors of the ECOs or BHCs must be much more massive (and larger in size) than those of the NSs. Then it follows from the magnetic flux conservation law that BHCs (at the galactic level) should have magnetic fields considerably higher than NSs. It is also probable even when they are old, their diminished magnetic fields are considerably higher than  $10^{10}\text{G}$ . In such cases, BHCs will not exhibit Type I X-ray burst activity. There may *indeed be evidence for intrinsic (high) magnetic fields* for the BHCs [3,4]. However, in some cases, they may well have sufficiently low magnetic field and show Type I bursts. It is now known that Cir X-1 which was considered a BHC, did show Type I burst, a signature of “hard surface”. Irrespective of interpretation of presently available observations, our work has shown that the BHCs cannot be, in a strict sense, (finite mass) BHs because then *timelike geodesics would become null* on their EHs.

Although future observations must settle questions about the true nature of BHCs, we remind the reader, that, *our results are exact subject to the validity of GTR*. Finally, if we demand that at sufficiently small scales quantum mechanics must take over GTR, the final state of continued ideal spherical collapse might be a Planck globule of mass  $M_{\text{pl}} \sim 10^{-5}\text{g}$ , irrespective of the initial value of  $M_i$ . We can only conjecture now whether the globule may or may not comprise wiggling elementary strings.

There have been many publications which claim that studies of gravitational collapse leads to formation of either BHs or naked singularities. And some readers may wonder why our result differs from so many studies. Here, first, it should be borne in mind that all such studies involve either semi-analytical or semi-numerical treatment. And since pursuance of singularity, by definition, involve handling of infinite values of various physical quantities, only, the results obtained by exact analytical treatments could be reliable. Again as asserted in the introduction, one must be able to handle the progress of the exact ever evolving EOS and radiation transport properties all the way upto the singularity in any analytical or numerical study in case that study requires specific form of EOS or radiation transport properties. Now obviously we cannot treat situations involving arbitrary large pressure, radiation density etc in an exact manner both because of theoretical and numerical limitations. Thus, in a strict and true sense, study of the problem of gravitational collapse of physical matter by any brute force direct method of “solving” is out of question. And if we take stock of the events, perhaps, (apparently) was the attempt by OS. And as we saw the result that trapped surfaces do not form is also hidden in Eq.(36) of their paper. And as mentioned earlier, the fluid considered by OS did not correspond to any physical fluid because all physical fluids have finite pressure and density gradient (under self-gravity). Finally the *must be allowed to radiate out* as it collapses otherwise its gravitational mass  $M$  would remain fixed and a trapped surface might appear.

Even then, we feel that if the authors of all such papers would (i) either try to find a global equations (5) and (7), or (ii) write an exact equation (such as Eq.36 of OS) connecting  $T$  with the Invariant circumference Radius (a scalar)  $R$ , they would arrive at the conclusion same as ours. In other words, before attempting to “solve” the collapse equations, if they would study their generic properties, they would also find that the final state would correspond to  $M = 0$ . And as pointed in Paper I, even though the previous authors overlooked such crucial generic properties, atleast in some cases there are broad or even exact agreements with our results. For instance

(i) Our result that there are no finite mass BHs is (apparently) in agreement with the finding that singularities could be “naked” and not finite mass BHs.

(ii) Our result that gravitational singularities should have  $M = 0$  is in agreement of the result of Lake [59]. As mentioned in Paper I, this is also in agreement with several other previous ideas [60,61].

(iii) Our assertion that realistic studies of gravitational collapse of physical matter must include pressure gradient is shared by many authors. Specifically, we would like to mention the work of Cooperstock, Jhingan, Joshi and Singh (1997) [62]. In this work, these authors pointed out that inclusion of pressure gradient forces is very important and formation of BH may be avoided by it. They also found that “the naked singularity arising in spherical collapse is necessarily massless in the sense that the mass function  $m(t, r)$  will go to zero at the naked singularity” in conformity with our Eq.(16) (their  $m$  is ours  $M$ ). Below Eq.(17) of this paper, these authors write that

“in the limit  $R \rightarrow 0$  we have  $m = R/2$ ”

in exact conformity with our result that the final state corresponds to a zero mass BH. But since the very concept of a BH generally implies  $M > 0$ , these authors called such a final state to be “marginally naked” rather than a Black Hole. In the context of studies of final state of spherical gravitational collapse, these authors correctly admit that, that “most of the results available thus far numerical in nature”.

One of the definitive signature of existence of finite mass BHs would be the evaporation of primordial BHs at the present epoch [63]. Gamma ray astronomers have been searching for these phenomenon for the last 25 years without any success. And we can predict with absolute certainty that no such events would ever be detected because GR does

not allow formation/existence of finite mass BHs.

If a BH would exist, the proper length of an infalling astronomer or anything would become  $\infty$  as it would approach the central singularity [23] because of infinite tidal gravity. Then how would the observer stay put in a geometrical point? There is no resolution for such inconsistencies within the BH paradigm. Such inconsistency actually does not arise because we have shown that (1) there cannot be any finite mass GR singularity for isolated bodies and (2) in case the astronaut is falling along with the collapsing matter, the proper radial length of not only the astronaut but that of the star too tend to become infinite. And there would be no problem in accommodating an infinitely stretched astronaut in the collapsing body.

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Note added in proof: After this paper was sent to press, Dr. Torres of Princeton Univ. pointed out that it is likely that the BHC at the galactic center and several other BHCs could be Bosonic stars. For all practical purposes the compactness of these bosonic stars could be like that of supposed BHs. Further, since both neutrino condensations or such bosonic condensations comprise weakly interacting particles, matter accreting onto them would hardly interact on impingement. Thus even though such condensations will have a physical surface, it may appear that matter is being lost through an “Event Horizon”.

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